Article

3-D Aerofoil Design with Increased Lift and Low Drag Profile

Vinod Kumar C[1], SC Gupta[2]

Abstract

This work aims at generation of optimum camber shapes for a wing for transport aircraft application. Induced drag reduction is the main consideration. The lift is increased by a factor and objective function for minimum induced drag is formed that is differentiable with respect to circulation that is aimed to generate prescribed lift.

Keywords: 3-D aerofoil design, Aircraft, Camber, Lift

Introduction

Moderately swept back wing is considered for the application in favor of medium aspect ratio and low induced drag. Design and analysis is done for Mach number of 0.7 and 4^0 angle of attack. Several parameterization techniques are seen in literature that have been developed and applied over the years for aerofoil design; such as polynomial representation and non-uniform rational B-splines (NURBS). These offer precision in representing and manipulating analytical curves [1, 2]. In our work, forces acting on the wing are determined through panel numeric. Downwash influence coefficients are determined within the framework of linearised potential flow theory. Principles of calculus of variations are applied to form objective function [3, 4].

Constraint optimization technique is applied to the definition of camber lines for minimum induced drag for the specified value of lift. Program is developed in Fortran language. Gfortran complier is used that is available in Fedora operating system of VMware which has Linux based platform. Following command complies and creates a binary ready for executions.

```
gfortran -o  program name  program  name.f
```

Following command executes the program file

```
./program name<input file name
```

Following command opens the program in vi editor

```
vi program.f
```

[1]PG Student, Final Year, [2]Professor and Head, Department of Aeronautical Engineering, MVJ College of Engineering, Bangalore, Karnataka, India.

Correspondence: Mr. SC Gupta, Department of Aeronautical Engineering, MVJ College of Engineering, Bangalore, Karnataka, India.

E-mail Id: satish_chander_gupta@yahoo.co.in

Orcid Id: http://orcid.org/0000-0003-2648-6611

Following commands have been used in the gfortran complier.

- pwd present working directory
- cd change directory
- mk dir create directory
- vi program.f opens the program in vi editor
- cpprog.f/directory name copy program
- :q quit editor mode
- se nu gives no sequence for entire file
- rm program name removes a program
- gfortran - o program name program name.f compiles and creates a binary ready for execution
- ./program name executes the program file

Encouraging results are obtained with this application.

Mathematical Modeling

Aerofoil shape parameterization can be obtained through definition of curves. NURBS is such a technique that utilizes a knot vector as defined below:

$$C(\xi) = \sum_{i=1}^{k} \frac{E_{i,n} W_i}{\sum_{j=1}^{k} E_{j,n} w_j} P_i \tag{1}$$

In this formula ξ is the curve parameter, ranging from 0 (the start of the curve) to 1 (end of the curve) , k is the number of control points, $E_{i,n}$ is the i-th basic function of order n, w_i is the weight associated with the i-th control point, and $P_i=[x_i,y_i]$ is the control point.

In the numerical optimization, the change of aerofoil curve shape is achieved by changing the coordinates of NURBS control points. Control point in this approach cannot move along a direction more than a defined length with prescribed limits. Limits are to be imposed on one of the NURBS control points. Shape-change framework with Radial basis function (RBF) is another approach for aerofoil optimization. Using the RBF method, the aerofoil doesn`t need to be parameterized, and is instead expressed as a cloud of points of arbitrary order and spatial resolution. There are a large number of basic functions to choose from. A radial basis function operates on radius between points, and returns a scalar value. The returned value will vary between 1.0 when the distance is 0, and 0 when the distance is equal to support radius. This support radius is chosen by the user, and represents the radius of influence of one point on other points. Camber changes occur through the camber cloud, which causes a change in the shape change cloud, which then in turn changes the external shape to reflect the change in camber. This essentially becomes trial and error technique.

In the approach made herein, wing is represented by a large number of constant pressure panels to model circulation. These panels are used for estimation of pressure difference coefficient, lift and induced drag. Only half of the wing is considered. Other half of wing is imaged. The program uses vortex panel method to calculate downwash by satisfying tangential flow conditions to determine singularity strength of each of panels; and thus lift, induced drag and moments for the given planform are determined [3, 4]. Control point for determining downwash is taken at 0.95% of local panel chord. The imaging of half of wing is done though the following expressions:

$$y = y_k - y_p \tag{2}$$

$$y(image) = -y_k - y_p$$

The subscript p refers to panel and subscript k refers to control point location. Panel circulation (γ) is determined through tangential flow boundary condition and pressure difference coefficient is given by $\Delta Cp = 2\gamma / U$. Lift is determined through integration of ΔCp over the chord and span. The program is developed in Fortran. It generates the output matrix of pressure difference coefficients from where lift, drag and moments are determined and then a optimization constraint of lift is introduced i.e., lift before and after the optimization remains same. Due to the effort of optimization, the given wing camber gets modified and hence a new pressure difference distribution results. Axis system and related moments are shown in Fig. 1. Half of wing is divided into nine chordwise and nine spanwise panels, making a total of 81 numbers of panels.
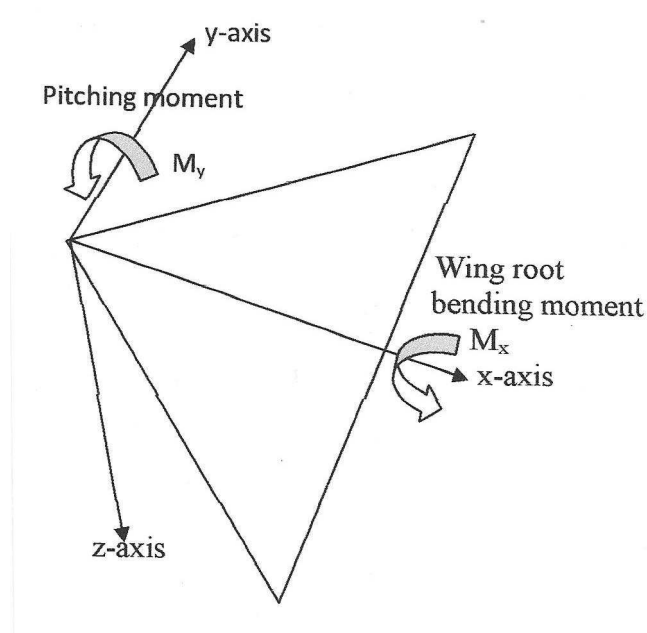
Figure 1.Axis System and Related Moments

Objective function (F) for the drag (D) minima and specified value of lift is written in the Lagrange form using Lagrange multipliers (λ_0):

$$F = D + \lambda_0(L - \bar{L})$$
(3)

Where $\bar{L}$ is the specified value of lift which is expressed in the following way:

$$\bar{L} = L \times LIF$$
(4)

Here L is the lift of a given profile that is to be increased by a factor LIF.

Lift is expressed in terms of circulation as below for N number of singularities.

$$L = \rho U\left[A_1 \gamma_1 + \cdots + A_N \gamma_N\right]$$
(5)

Here. $\gamma_1 \dots \gamma_N$ are circulation strength of N number of panels and $A_1, \dots A_N$ are related panel areas. Lift effects are simulated by a spread of vortex sheet (circulation strength). The property of the vortex sheet is that the component of flow velocity tangential to sheet experiences a discontinuity change across the sheet that is given by:

$$\gamma = u_1 - u_2$$

Where u_1 and u_2 are tangential velocities just above and below the sheet respectively.

Matrix of optimization Eq. (6) below is obtained by differentiation of objective function (F) w.r.t circulation γ.

$$\begin{bmatrix} 2A_1 a_{1,1} & . & . & . & . & (A_1 a_{1,N} + A_N a_{N,1}) & A_1 & A_1 XC_1 \\ . & & & & & . & . & . \\ . & & & & & . & . & . \\ . & & & & & . & . & . \\ (A_N a_{N,1} + A_1 a_{1,N}) & . & . & . & . & 2A_N a_{N,N} & A_N & A_N XC_N \\ A_1 & . & . & . & . & A_N & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \gamma_1 \\ . \\ . \\ . \\ \gamma_N \\ \lambda_0 \end{bmatrix} = \begin{bmatrix} 0 \\ . \\ . \\ . \\ 0 \\ L \times LIF \end{bmatrix}$$
.... (6)

Where $XC_1, \ldots XC_N$ are distances of panel control points from wing apex; and $a_{i,j}$ are influence coefficient towards downwash velocities.

Solution of this matrix for prescribed lift $\overline{L} = L \times LIF$ values results in optimal circulation from where pressure difference coefficient, drag, and wing root bending moment etc. are determined [5].

Results and Discussions

Following wing is taken as candidate for study. Flow Mach number is taken as 0.7 and angle of attack is taken as 4^0. Wing has practical application for high speed transport aircraft and as well as for UAVs. The value of LIF is taken as 1.2 for increasing the lift.

Root chord = 2.9m

Tip chord = 1.2909m

Semispan=7.99m

Leading edge sweep = 0.349 radians

Trailing edge sweep = 0 .16 radians

Flat surface wing shape is first considered and is subjected to optimization towards minimum induced drag. The lift is then incremented by a factor and again the optimal profile for minimum drag is worked out. Table 1 below shows the aerodynamic data comparison for the case of lift incremented wing with the base line wing profile.

Table 1 shows that there is substantial potential for drag reduction through optimization. Figure 2 shows the values of chordwise pressure difference coefficient before and after optimization at root station for Case-A. There is shift in local lift towards the mid of chord. Similar feature is seen towards the tip of wing (Fig. 3). This is the reason for the increase in pitching moment ratio value. The increase in pitching moment is more in Case-B, since there is increase in local lift towards trailing edge. There is also shift in local loading towards tip as seen by the values of ratio of wing root bending moment before and after optimization (Case-A, value of 1.0226 and Case-B, value of 0.8522).

Table 1.Comparison of Aerodynamic Coefficient

Conditions	C_L	Before optimization C_D	After optimization C_D	% Drag reduction	M_x / M_x^*	M_y / M_y^*
Without incremental value of lift (Case-A)	0.381	0.0266	0.0093	65%	1.0226	0.817
After incremental value of lift by a factor of 1.2 (Case-B)	0.457	0.0319	0.0134	58%	0.8522	0.6813

Pressure difference coefficient for the Case-B is shown in Fig. 4 for two stations. There is similarity of shift of local lift with the Case-A. In both the cases, there is shift in location of adverse pressure gradient. The shift is towards the rear, and this is favorable from the point of delay in flow separation.
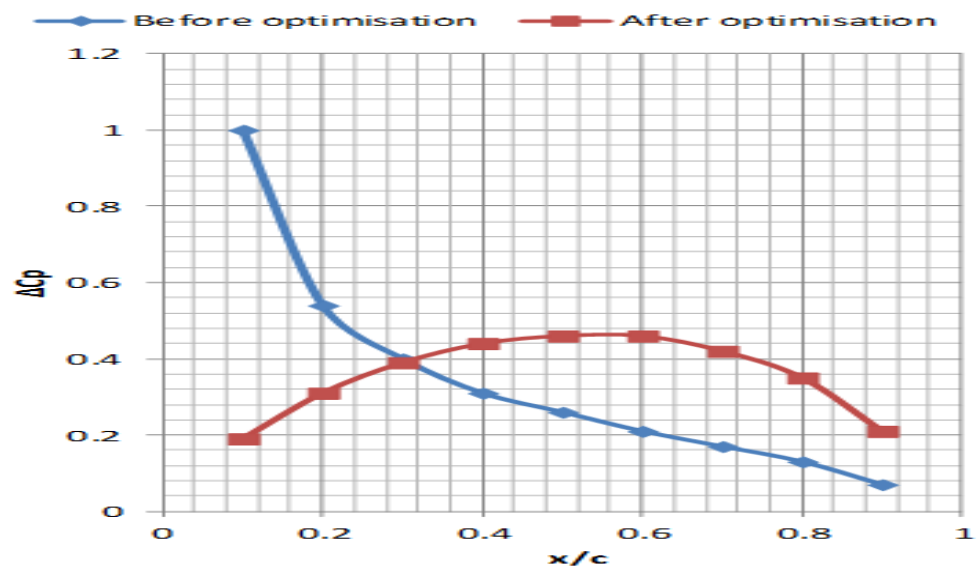
Figure 2.Pressure Difference Coefficient at Root Station without Lift Increment (Case-A)
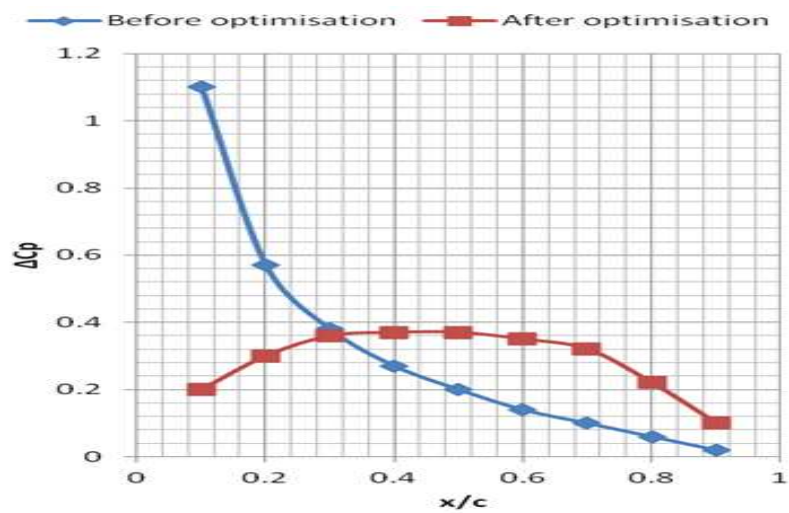

Figure 3.Pressure Difference Coefficient at Tip Station without Lift Increment (Case-A)
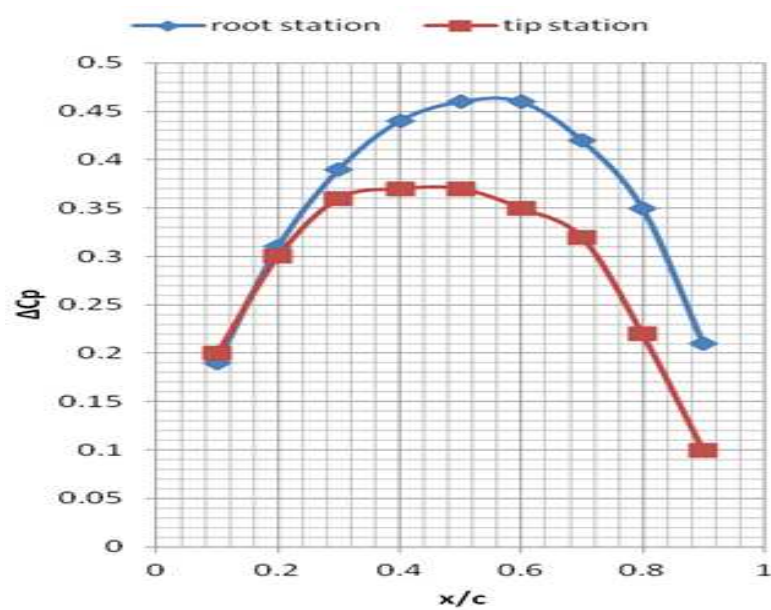

Figure 4.Pressure Difference Coefficient with Lift Increment (Case-B)
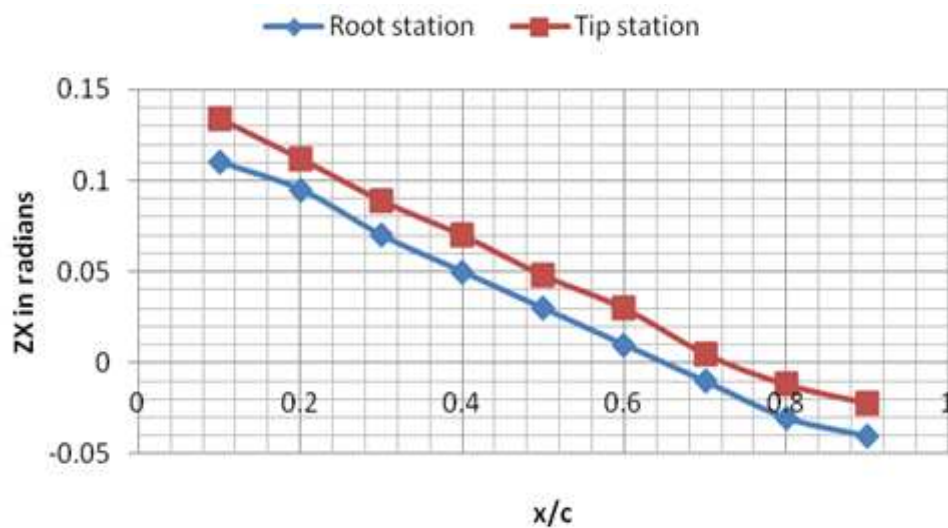
Figure 5.Camber in Radians after Optimization before Lift Increment (Case-B)
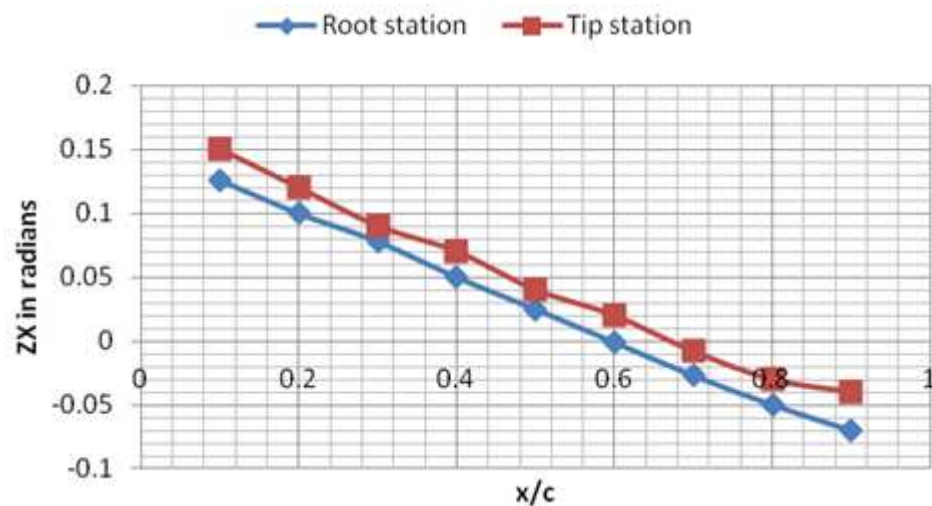

Figure 6.Camber in Radians after Optimization after Lift Increment (Case-B)

Resulting optimal camber is shown in Fig. 5 and 6 in the chordwise manner (note that the initial camber in both the cases is nil, i.e. a flat surface case). The requirement of spanwise twist is lesser in Case-B. This is visible by the lesser displacement of camber values from root to tip.
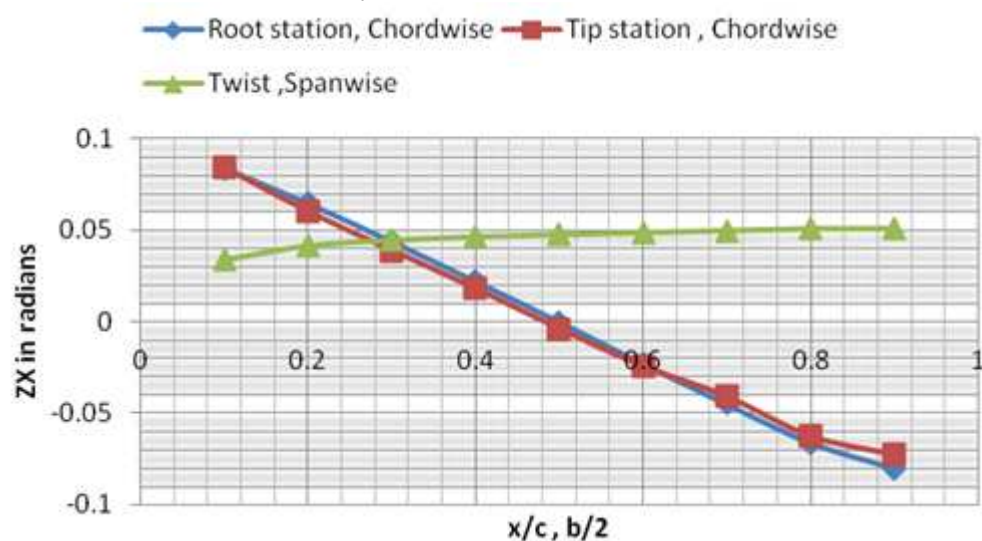

Figure 7.Twist Separated from Camber, Case-B

The optimal warp of Case-B is separated for camber and twist by a using a subroutine given at Appendix `A`. Resulting values of twist and camber are shown in Fig. 7. The resulting aerofoil camber is seen to be same at root and tip. This is very advantageous from manufacturing ease point of view. The twist is also nearly linear.

Excellent features of design are obtained by ths technique followed herein.

Values of % camber in chordwise manner are worked out and shown in Fig. 8. A single aerofoil results at root as well at tip across the span after separation of twist.
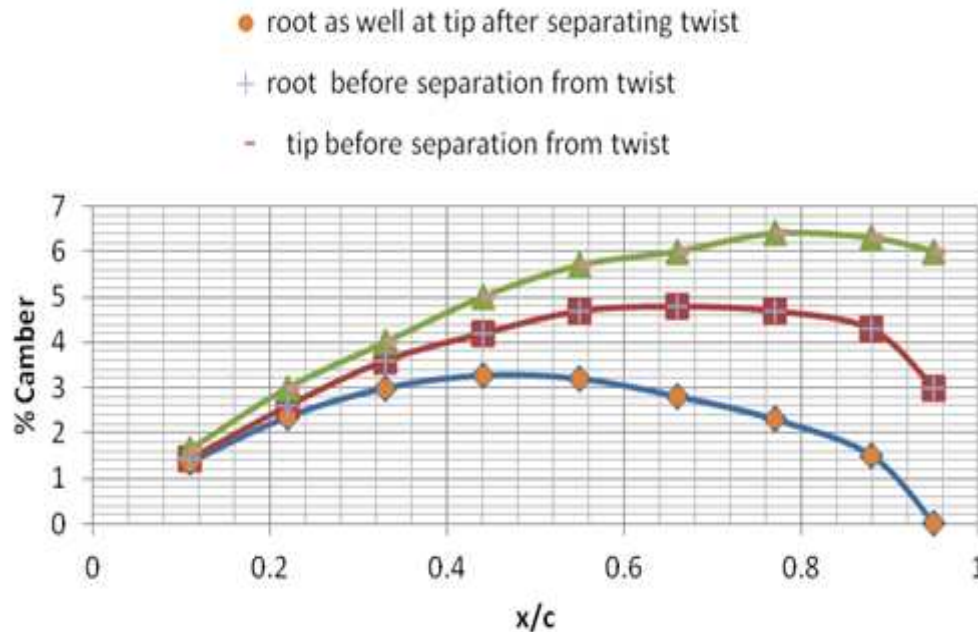

Figure 8.Values of % Camber in Chordwise Manner

References

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Appendix `A`

```
C WING WARP IS BROKEN INTO TWIST AND AEROFOIL CAMBER

C N=NUMBER OF PANELS, XP= PANEL CORNERS , RT=TWIST

C AZ= GIVEN WARP TO BE BROKEN INTO AEROFOIL CAMBER + TWIST

SUBROUTINE TWISTR(XP,ZZ,RT,AZ,CAMBER,N)
DIMENSION XP(10,10),ZZ(10,10),RT(10),AZ(9,9)
DIMENSION XZZ(10,10),CAMBER(9,9)
DATA IR,IW/5,16/
N1=N+1
DO 100 J=1,N
DO 150 I1=1,N1-1
I=N-I1+1
DIS=XP(I+1,J)-XP(I,J)
150 ZZ(I,J)=ZZ(I+1,J)-DIS*AZ(I,J)
100 CONTINUE

WRITE(IW,200)
200 FORMAT(/2X,'VALS OF THE VER-ORDI ROWS READ CHORDWISE')
WRITE(IW,225)((ZZ(I,J),I=1,N),J=1,N)

DO 250 J=1,N
250 RT(J)=-(ZZ(1,J)-ZZ(N+1,J))/(XP(N+1,J)-XP(1,J))

WRITE(IW,275)
275 FORMAT(/2X,'VALS OF THE SPANWISE TWIST DISTRIBUTION')

WRITE(IW,225)(RT(J),J=1,N)
DO 300 J=1,N
DO 350 I=1,N

350 CAMBER(I,J)=AZ(I,J)-RT(J)
300 CONTINUE
WRITE(IW,375)
375 FORMAT(/2X,'VALS OF THE AEROFOILS CAMBER DIST')
WRITE(IW,225)((CAMBER(I,J),I=1,N),J=1,N)
DO 660 J=1,N
XZZ(I,J)=0.
DO 677 I=2,N
677 XZZ(I,J)=XZZ(I-1,J)+CAMBER(I,J)*(XP(I+1,J)-XP(I,J))

660 CONTINUE
DO 899 I=1,N
DO 899 J=1,N

899 XZZ(I,J)=XZZ(I,J)/(XP(N+1,J)-XP(1,J))

WRITE(IW,775)
775 FORMAT(/2X,'VALS OF AEROFOIL NON-DIM

1VER-ORDI ROWS READ CHORDWSE')
WRITE(IW,225)((XZZ(I,J),I=1,N),J=1,N)
225 FORMAT(9F8.4)
STOP
END
```

Appendix `B`

Nomenclature

A = Panel Area

$a_{i,j}$ = Influence coefficients (Influence of j_{th} panel on i_{th} control point)

b = Span

c = Local chord length

C_D = Drag coefficient

C_L = Lift coefficient

ΔCp = Pressure difference coefficient

D = Induced drag

L = Lift

LIF = Lift Increment Factor

M = Mach number of freestream

N = Total number of panels on half of wing

M_x = Wing root bending moment before optimization

M_x^* = Wing root bending moment after optimization

M_y = Pitching moment before optimization

M_y^* = Pitching moment after optimization

x, y, z = Cartesian coordinates in geometric plane ; x is chordwise, y is spanwise and z is vertical

XC = x-Distance of panel control point from the apex of wing

dz/dx = Aerofoil camber slopes (ZX)

U = Freestream velocity

α = Angle of attack

$\beta = \sqrt{\left| 1 - M^2 \right|}$

γ = Circulation

ρ = Density

Suffix

i = i_{th} Panel control point

j = j_{th} Panel